## FORMS IV and V

**Unit 1**

| Unit No. | Basic Content/Objectives | Detailed Content | | Time Ratio | Notes on Teaching |
|---|---|---|---|---|---|
| 1 | **More about quadratic equations: surds**<br><br>*Objectives:*<br>(1) To acquire skills in solving quadratic equations by completing the square and by using formula.<br>(2) To solve simultaneous equations, one linear and one quadratic.<br>(3) To learn the relation between roots and coefficients.<br>(4) To learn the rationalization process. | 1.1 | Completing the square. | 3 | By now students are quite familiar with the various techniques of solving equations including quadratic ones; using graphical or factor methods. Now they are introduced to a skill that requires thorough understanding of algebraic operations. Teachers should begin with examples like $x^2-8x+9=0$ and progress to examples like $3x^2-6x+4=0$, where the coefficient of $x^2$ is not unity. The steps in completing squares can be summed up on the board for ease of reference but students need not memorize the steps. |
| | | 1.2 | Formula. | 5 | Once the students understand how the formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ for solving $ax^2+bx+c=0$ is derived, they should be able to reproduce it when needed. Teachers should ensure that students have no difficulty in applying the formula. One useful hint to see whether students know how to apply the formula is to ask them to write down the values of *a*, *b* and *c* first before they attempt to substitute them into the formula.<br>When the students are quite familiar with the different techniques of solving quadratic equations, teachers may then ask if they could see any relations between the sum, product of the roots α, β and the coefficients of $ax^2+bx+c=0$. Then the relations $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$ should be introduced and proved. Exercises on calculating the values of expressions such as $\frac{1}{\alpha}+\frac{1}{\beta}$, $\alpha^2+\beta^2$, $\alpha^3+\beta^3$ and exercises in the formation of quadratic equations should be included. |
| | | 1.3 | Simple problems using quadratic equations. | 4 | Problems requiring the solution of a quadratic are numerous in many school texts. Teachers should select those that have relation to students' experiences and preferably, have bearing on the practical application of mathematics. Examples can also be taken from physics or chemistry courses. |



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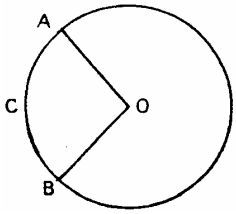
**Unit 1**

| Unit No. | Basic Content/Objectives | Detailed Content | | Time Ratio | Notes on Teaching |
|---|---|---|---|---|---|
| 1 | | 1.4 | Simultaneous equations: one linear and one quadratic. | 4 | It is desirable first to solve simultaneous equations in which one is linear and one is quadratic by using the graphical method. The graph of the quadratic should be plotted first and when a suitable straight line graph is added, the solutions may be readily obtained. Examples should be so chosen that one quadratic graph is used repeatedly to solve many quadratic equations. This will save students' time in plotting too many quadratic graphs. Teachers may find the graphical method useful in explaining why some quadratic equations have two roots, one root or no root at all.<br>The algebraic method of substituting the linear equation into the quadratic equation should then be introduced and sufficient demonstration and practice should follow to ensure complete mastery of the technique. |
| | | 1.5 | Rationalization of surds. | 6 | When an equation such as $x^2+3x-1=0$ is considered, it is natural to leave the answers in surd form. The term "surd" can then be explained and students are expected to be able to transform surd of any order into a surd of a different order. Addition, subtraction, multiplication and division of surds should be practised thoroughly before introducing the process of rationalizing the denominators of expressions of the form $\frac{1}{\sqrt{a}\pm\sqrt{b}}$. |

## FORMS IV and V

**Unit 2**

| Unit No. | Basic Content/Objectives | Detailed Content | | Time Ratio | Notes on Teaching |
|---|---|---|---|---|---|
| 2 | **Basic properties of a circle**<br><br>*Objectives:*<br>(1) To acquire an informal treatment of geometric argument.<br>(2) To learn the basic properties of a circle, tangents to a circle, cyclic quadrilateral, and the tests for concyclic points. | | General approach. | | In this unit, students will be expected to justify, follow and understand each deductive step of a proof, but no attempt should be made to build a formal and rigorously deductive structure based on carefully specified postulates and axioms. Students should not necessarily be expected to reproduce a formal proof of a geometric theorem. |
| | | 2.1 | Chords and arcs of a circle. | 5 | The meaning of terms such as “arc”, “segment”, “sector” and “chord” should be reviewed. In order to differentiate between major and minor arcs, segments and sectors, it is simpler just to use an extra letter on the diagram.<br>  <br><br>e.g. Use $\widehat{ACB}$ rather than “minor $\widehat{AB}$” and sector *OACB* rather than “minor sector *OAB*”<br>Teachers may emphasize that radii and chord form an isosceles triangle, and may use congruent triangles to show that the perpendicular to a chord from the centre of a circle bisects the chord. The fact that equal chords are equidistant from the centre follows. |
| | | 2.2 | Angles in a circle. | 10 | In this sub-unit we are concerned with the angle at the centre, the angle at the circumference, angle in a semi-circle and angles in a cyclic quadrilateral. |



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**Unit 2**

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| 2 | | | | | This work may be made a little more interesting if the central symmetry of a circle is used. An overhead projector is invaluable for this kind of demonstration. Alternatively use tracing paper, a pin and revolve the tracing paper to show that equal arcs subtend equal angles at the centre. This is also true for chords, but teachers may wish to validate the previous method by showing that "equal angles at the centre are subtended by equal chords". This may also be demonstrated using congruent triangles. However, it should be emphasized that, whereas arcs are proportional to the angles they subtend at the centre, chords are NOT.<br>When demonstrating (the three cases) that the angle at the centre is twice the angle at the circumference and the angles in the same segment subtended by the same arc are equal, it is helpful for students to actually see that as the vertex of the subtended angle moves round the circle, the angle remains the same size. This may be done using a simple piece of apparatus such as a bead running on a wire arc and held by elastic bands. A piece of card can be used for the angle in order to show that it remains a constant size. |
| | | 2.3 | Properties of cyclic quadrilateral and the tests for concyclic points. | 11 | The properties of cyclic quadrilateral such as<br>(a) the opposite angles of a cyclic quadrilateral add up to 180°, and<br>(b) if one side of a cyclic quadrilateral is produced, the exterior angle so formed is equal to the interior opposite angle<br>should be proved. Sufficient exercises soluble by these properties should be given.<br>The converses of the above properties constitute two tests for four concyclic points. It is also known that if the straight line joining two points subtends equal angles at two other points on the same side of it, then the four points are concyclic.<br>These three tests for concyclic points should be discussed thoroughly and proved in detail. |

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**Unit 2**

| Unit No. | Basic Content/Objectives | Detailed Content | | Time Ratio | Notes on Teaching |
|---|---|---|---|---|---|
| 2 | | 2.4 | Tangent to a circle at a point and angles in the alternate segments. | 11 | The compasses construction forming the right angle should be done to emphasize the perpendicular property. However, in general, it is sufficient for students just to lay a ruler against the circle, at the point, in order to draw the tangent. Students are expected to know the basic properties of tangents.<br>On completion of the teaching of angles in the alternate segments students should be exposed to an extensive array of miscellaneous exercises that make use of all the geometry done so far. |
| | | 2.5 | A circle passing through three non-linear points. | 3 | The construction follows from the corollary to 2.1 para. 2, i.e. the centre of a circle lies on the perpendicular bisector of a chord. This construction not only emphasizes the theorem but is also another way of looking at the circumscribed circle of a triangle. The limiting case where the three points are collinear may be of interest to abler students. |
| | | | | 40 | |



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**Unit 3**

| Unit No. | Basic Content/Objectives | Detailed Content | | Time Ratio | Notes on Teaching |
|---|---|---|---|---|---|
| 3 | **Functions**<br><br>*Objectives:*<br>(1) To recognize the different kinds of numbers.<br>(2) To understand the basic idea of a function.<br>(3) To learn how to use the notation for a function.<br>(4) To manipulate polynomials. | 3.1 | Number systems: integers, rational numbers, irrational numbers and real numbers. | 5 | This is essentially a revision of the elementary properties of integers and fractions and students are introduced to a new term: "rational numbers". An appropriate explanation of a rational number should be given according to the ability of the class. The characteristics of rational numbers when expressed in decimals should be demonstrated.<br>e.g. Terminating decimal $\frac{2}{5} = 0.4$<br>$\frac{-3}{1} = -3.0$<br>Recurring decimal $\frac{1}{3} = 0.\dot{3}$<br>$\frac{2}{7} = 0.\dot{2}8571\dot{4}$<br>However, it should be noted that irrational numbers,<br>e.g. $\sqrt{2}$, $-\sqrt{6}$, $\sqrt[3]{9}$, $\pi$, do not behave in this manner.<br>The sets of rational and irrational numbers form the set of real numbers. Detailed and in-depth discussion of the real number system is NOT necessary. |
| | | 3.2 | Concept of a function. | 4 | The idea of a function can be introduced as a relation between two varying quantities. Teachers may find that the idea of the number producing machine gives a useful pictorial representation in this context. However, students should see that functions transform numbers rather than generate them. Teachers should be sure that students do not try to solve functions as if they were equations.<br>Teachers should give more examples of function such as $\sin x°$, $\cos x°$ and $\log x$, etc. |

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**Unit 3**

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 3 | | 3.3 Notation for a function: $f(x)$ and $y = f(x)$. | 4 | The notation $f(x)$ should be introduced first and then a suitable letter, such as *y*, is introduced to denote $f(x)$. In this way students can see that a function may be represented graphically on a coordinate plane.<br>Once students are familiar with the notation, they may be asked to attempt questions like:<br>Given $f(x) = x^2 + 2x - 1$, what are $f(0)$, $f(-2)$, and $f(a-1)$? |
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## FORMS IV and V

**Unit 4**

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 4 | **More about polynomials**<br>*Objectives:*<br>(1) To acquire skills in factorizing polynomials by factor theorem.<br>(2) To find the H.C.F. and L.C.M. of polynomials.<br>(3) To manipulate algebraic fractions. | 4.1 Manipulation of polynomials. | 4 | At this stage, it is desirable to revise the manipulation of polynomials. Addition, subtraction and multiplication of polynomials are standard work. Teachers may like to demonstrate that division of one polynomial by another does not generally lead to a polynomial. In preparation for further work, students should see and be able to recognize the general polynomial written in the form<br>$a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$. |
| | | 4.2 Remainder theorem and factor theorem. | 4 | Teachers are simply expected to illustrate the remainder theorem using a quotient and divisor notation such as $f(x) = (x-a)Q(x) + f(a)$. To consolidate the idea, students may also verify this theorem using the long division method. Then the factor theorem can be deduced. |
| | | 4.3 Factorization by factor theorem. | 9 | The use of the factor theorem becomes apparent when there is a need to factorize polynomials of degree three or higher. Students should also see that factorization will lead to the solution of the equation $f(x) = 0$. Functional notation should be used and the technique of using detached coefficients and synthetic division may be introduced. Questions of various types should be used to test thorough understanding and mastery of the factorization process.<br>In using the factor theorem to factorize the polynomial $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$, where $a_0$, $a_1$, $\cdots$, $a_n$ are integers, it is necessary to obtain a number $\alpha$ so that $f(\alpha) = 0$. A primitive method of getting $\alpha$ is by trial and error. When $\alpha$ is a rational number $\frac{p}{q}$, teachers should discuss the conditions for $px - q$ to be a factor and deduce the relation between $p$, $q$, $a_0$ and $a_n$, In order to have a factor $px - q$, some rules have to be developed which serve as a better method of factorizing $f(x)$. |

**Unit 4**

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 4 | | | | By factorizing $x^3+1$ and $x^3-1$, students may discover the identities $x^3 \pm 1 \equiv (x \pm 1)(x^2 \pm x + 1)$ and will accept the generalized result of $x^3 \pm y^3 \equiv (x \pm y)(x^2 \pm xy + y^2)$ readily. |
| | | 4.4 H.C.F. and L.C.M. | 3 | The idea of finding H.C.F. and L.C.M. of integers may be revised and analysed. This immediately leads to the factorization method of finding H.C.F. and L.C.M. of polynomials. Emphasis should be laid upon the factorization method and other methods may be excluded. |
| | | 4.5 Manipulation of simple fractions. | 5 | Students are expected to master the technique of manipulating simple fractions using the four rules. Teachers may wish to find the L.C.M. of polynomials as a prerequisite to this topic; a direct manipulation of these fractions is also effective if done skillfully. It is advisable, therefore, to show a variety of examples that direct the students to the techniques of simplification rather than to the skill in manipulating long algebraic expressions. |
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**Unit 5**

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 5 | **Proportion and variation**<br><br>*Objectives:*<br>(1) To acquire further knowledge in rate, ratio and proportion.<br>(2) To practise more in the use of rate, ratio, proportion and variation. | 5.1 More on rate, ratio and proportion. | 4 | This is an extension of Unit 1 in Form II whence students were given the meaning of rate, ratio and proportion. However, students should make clear that rate provides a comparison of quantities not of the same kind and it bears a unit such as km per hour, while ratio compares quantities of the same kind and hence bears no unit. Students should be pointed out that ratio serves a better comparison between two quantities than using their difference. For example 10 is less than 20 by 10 just as 990 is less than 1 000 by the same amount. However, using ratio, one can have a better view.<br>Since students have already seen some examples on rate, ratio and proportion, questions like in what ratio certain mixture of spirit could be mixed with water so as to decrease the percentage of spirit in the original mixture can be discussed. Students should make clear the idea of inverse ratio and hence its application. In problems on rate of working, the idea of treating the job as a unit quantity should be introduced for the manipulation. |
| | | 5.2 Algebraic manipulation of ratio and proportion. | 5 | Basic rules for ratio and proportion should be discussed and proved. For example if $\frac{a}{b}=\frac{c}{d}$, then $ad = bc$ and so on. Afterwards, ideas can be extended to continuous proportion, i.e. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\cdots$, then each is equal to $\frac{ka+mc+ne+\cdots}{kb+md+nf+\cdots}$ where $k, m, n, \ldots$ are constants.<br>If it helps, numerical values could be used to show the equality. For example since $\frac{2}{4}=\frac{3}{6}$ one can see easily that $2 \times 6 = 3 \times 4$。 |

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**Unit 5**

| Unit No. | Basic Content/Objectives | Detailed Content | | Time Ratio | Notes on Teaching |
|---|---|---|---|---|---|
| 5 | | 5.3 | Direct and inverse variation. | 5 | Students should make clear that variation refers to the change of certain quantity as some of its related quantities are changed. They should see that the change is regular and follows certain rule. Idea of dependence and independence can be shown by concrete examples such as the extension of spring with its acting loads.<br>Examples like the payment of bus fares shows the idea of direct variation and the sharing of a box of chocolates among some children shows the idea of inverse variation. The corresponding graphs of these two types of variation should be sketched and discussed. Special attention should be drawn upon the specific slopes of these graphs and hence a means to determine the variation constant. |
| | | 5.4 | Joint and partial variation. | 7 | Examples in science like the related change in the volume, pressure and absolute temperature of an ideal gas shows the idea of joint variation. On the other hand, the cost for making school badges with respect to the total number made illustrates the idea of partial variation. Many such examples in science and everyday life could be put forward to motivate students. |
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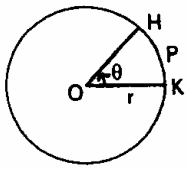
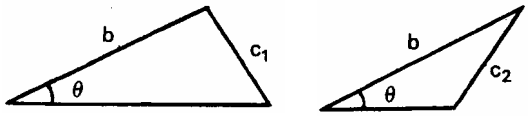
## FORMS IV and V

**Unit 6**

| Unit No. | Basic Content/Objectives | Detailed Content | | Time Ratio | Notes on Teaching |
|---|---|---|---|---|---|
| 6 | **More about trigonometry**<br><br>*Objectives:*<br>(1) To learn the concept of circular measure.<br>(2) To learn the functions sine, cosine and tangent in the interval 0 to $2\pi$, i.e. 0° to 360°.<br>(3) To solve easy trigonometric equations.<br>(4) To learn the area formula and the sine and cosine formulae of a triangle.<br>(5) To learn the techniques of solving triangles. | 6.1 | Measurement of angles in radians. | 2 | Students should understand the meaning of a radian and the need of introducing it for use in further mathematics. |
| | | 6.2 | Arc length and area of sector. | 2 |  <br>Students have already learned the ratio method to find the arc length and area of sector. Now they should derive the formulae:<br>$\widehat{HPK} = r\theta$ , where θ is in radian measure. Area of sector HOKP = $\frac{1}{2}r^2\theta$ where θ is in radian measure. |
| | | 6.3 | The functions sine, cosine, tangent and their graphs in the interval 0 to $2\pi$, i.e. 0° to 360°. | 4 | In defining the functions sine, cosine and tangent in the interval 0 to $2\pi$, i.e. 0° to 360°, teachers may find it useful to use coordinates. Mnemonics and formulae may be used provided students can work out the trigonometric ratios of any angles or formulae from first principles. This is particularly important as students may use electronic calculators.<br>In drawing graphs from 0 to $2\pi$, i.e. 0° to 360°, students may find it useful to choose the scale at intervals of $\frac{\pi}{6}$ i.e. 30°. Teachers can show students how and where the tangent graphs approach infinity. |
| | | 6.4 | Easy trigonometric equations (solutions in the interval 0 to $2\pi$, i.e. 0° to 360°). | 6 | At this stage, the solution of trigonometric equations is best illustrated by examples. Initially some simple trigonometric functions may be presented graphically and the students led to discover the solutions of trigonometric equations from them. After some practice, students should be taught to use tables to solve trigonometric equations, including quadratic equations which are factorizable. They are expected to be able to give all solutions in the interval 0 to $2\pi$, i.e. 0° to 360°. |

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 6 | | | | Although it is obvious that there is no limit to the number of possible solutions, the general solution of a trigonometric equation need not be considered at this level. |
| | | 6.5 Area of triangle as $\frac{1}{2}bc\sin A$. | 2 | The formula is true for any two sides and the included angle. It can also be demonstrated that the formula is true for both acute and obtuse angles. |
| | | 6.6 The sine and cosine formulae of a triangle. | 10 | It is not difficult for students to see how the sine and cosine formulae are derived.<br>Knowledge of the previous sub-unit can be used to derive the sine formula.<br>When it comes to the ambiguous cases, that is, two sides and one non-included angle, teachers should explain with the help of separate diagrams such as<br>Case (I) Case (II )<br>  <br><br>The cosine formula may be derived from the Pythagoras' Theorem or from the following three identities.<br>$a = b\cos C + c\cos B$<br>$b = a\cos C + c\cos A$<br>$c = b\cos A + a\cos B$ |





| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
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| 6 | | | | It is worthwhile to note that the Pythagoras' Theorem is a special case of the cosine formula.<br>It should be noted that the sine formula and the cosine formula together are sufficient to solve any triangle provided enough sides and angles are given to fix the triangle. Students should study elementary applications of these two formulae. |
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## FORMS IV and V

**Unit 7**

| Unit No. | Basic Content/Objectives | Detailed Content | | Time Ratio | Notes on Teaching |
|---|---|---|---|---|---|
| 7 | **Arithmetic and geometric progressions**<br><br>*Objectives:*<br>(1) To recognize A.P. and G.P.<br>(2) To learn the use of Σ notation.<br>(3) To learn some properties of A.P. and G.P.<br>(4) To learn the summation of A.P. and G.P. | 7.1 | Sequence and series. | 2 | Through the recognition of number patterns, students generally have no difficulty in understanding the meaning of sequence which simply means a string of numbers, sometimes with an easily recognizable pattern. However, students often find it difficult to define what a series is. It is suggested not to give a formal definition of series. More examples, especially of numerical type, should soon make the point clear. As a follow up to this topic, teachers may discuss some special number patterns such as triangular numbers, square numbers, rectangular numbers, etc. In-depth treatment of these patterns, however, should be avoided. The meaning of general term of a sequence should also be discussed and students are expected to know how to write down the first few terms of a sequence when the general term is given. |
| | | 7.2 | A.P. and G.P. | 4 | Students should be able to recognize these two types of progressions and also be able to write down the general terms when the progressions are given. After enough practice, students may consider progressions in which the constant increment or multiplier is negative, fractional or the square root of a certain number, etc. When a few terms of a progression are given, students should also know how to insert any number of terms between two given terms. |
| | | 7.3 | Summation notation. | 3 | As a preparation for the study of summation problems of A.P. and G.P., teachers may find it useful to first introduce the notation $\sum_{i=1}^{n} x_i$ . For the sake of abbreviating the notation further, it may be reduced to $\sum x_i$ and even to $\sum x$ . It can be seen that in actual manipulation, $\sum x$ is easier and more convenient to handle than others, provided no confusion arises. |



## FORMS IV and V

**Unit 7**

| Unit No. | Basic Content/Objectives | Detailed Content | | Time Ratio | Notes on Teaching |
|---|---|---|---|---|---|
| 7 | | | | | Properties such as<br>(a) $\sum(ax \pm by) = a\sum x \pm b\sum y$<br>(b) $\sum(x \pm y)^2 = \sum x^2 \pm 2\sum xy + \sum y^2$<br>are useful in Unit 8. They may be introduced but proofs are not required at this stage. |
| | | 7.4 | Summation of AP. and G.P. | 5 | The summation formulae may be derived using the Σ notation or other methods but students are not expected to reproduce the proofs. Practical examples should then be considered. The case of infinite G.P. may be briefly discussed and illustrated by examples. Thorough treatment of infinite series is not expected. |
| | | | | 14 | |

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 8 | **Probability and statistics**<br><br>*Objectives:*<br>(1) To learn more about probability and statistics.<br>(2) To apply the basic laws in probability to simple problems.<br>(3) To learn the concept and measures of dispersion of a distribution.<br>(4) To learn some applications of the mean and the standard deviation. | 8.1 Addition and multiplication laws. | 7 | This sub-unit is a continuation of Unit 9 in Form III. After students are acquainted with the idea of probability, it is natural for them to consider more complicated problems. Examples like tossing coins, throwing dice and drawing cards illustrate the concepts of "mutually exclusive" and "independent events". It would be harmful to try to give a formal definition at this stage. The addition law may be illustrated by numerical examples.<br>The concept of the multiplication law may be interpreted as a fraction of a probability. Initially, it is desirable for students to tackle problems from the common sense point of view, finally teachers should summarize the techniques in the form of the above two laws. For demonstration, numerical examples should not only involve theoretical probabilities but also experimental probabilities. For example:<br>The probability of having a boy in 0.55. What is the probability of having a boy and a girl? |
| | | 8.2 Weighted averages. | 6 | The mean of a frequency distribution of Unit 11 in Form III is in fact a mean weighted by its frequencies. It may be regarded as a weighted average (mean). The concept of weighted average is commonly known in practical life. Teachers should find no difficulty in collecting examples in daily life to illustrate the application of weighted average.<br>*Example:*<br>The following is the examination report of two students $T_1$ and $T_2$, |

| | $T_1$ | $T_2$ | No. of periods per week |
|---|---|---|---|
| Chinese | 70 | 90 | 10 |
| English | 65 | 60 | 8 |
| Maths. | 85 | 61 | 5 |





| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
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| 8 | | | | Which student did better in the examination?<br>In order to answer such a question we consider their average scores.<br>The average score of $T_1$ = $\frac{70+65+85}{3}$<br>= 73.33<br>The average score of $T_2$ = $\frac{90+60+61}{3}$<br>= 70.33<br>We say that $T_1$ did better than $T_2$.<br>On the other hand, if we take the weighted averages with the number of periods per week as weights then<br>the weighted average of $T_1$ = $\frac{70\times10+65\times8+85\times5}{10+8+5}$<br>= 71.52<br>the weighted average of $T_2$ = $\frac{90\times10+60\times8+61\times5}{10+8+5}$<br>= 73.26<br>We then say that $T_2$ did better than $T_1$.<br>Can we have other conclusions? |
| | | 8.3 Measures of dispersion: range, mean deviation, variance and standard deviation (grouped and ungrouped data). | 6 | In statistics, we accept that a set of data cannot be perfectly uniform. Teachers may use the following examples for illustration:<br>(a) the weights of children having the same height;<br>(b) the measurements of the diameter of a circle by different children;<br>(c) the heights of the ceiling as estimated by the students in the class;<br>(d) the monthly expenditure of families of the same size;<br>(e) the prices of the same brand of rice in different shops.<br>(Students may be asked to collect the date themselves so that they may have a better feeling of the variability.) |

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
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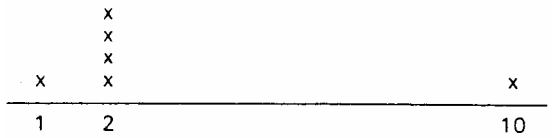
| 8 | | | | To measure the variability (dispersion) of a set of data, we may use the range, the mean deviation, the variance .or the standard deviation.<br>The simplest measure of dispersion is the range, which is defined as the difference between the largest and the smallest values in a set of data. The disadvantage of this measure is that it does not take the intermediate values into account. Thus, the following distributions |





have the same range, but certainly not the same dispersion.

A better measure is the mean deviation $= \frac{1}{n}\sum f\,|x-\bar{x}|$

The mean deviation directly gives the average difference of each number from the mean. Teachers should give a full explanation (including the absolute sign) as to how the mean deviation can measure the dispersion of the distribution. However, as the absolute sign is very difficult to handle in mathematical computation, we consider the

variance $= \frac{1}{n}\sum f\,(x-\bar{x})^2$ .



| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 8 | | | | This is also regarded as a measure of dispersion. The idea of squaring the difference from the mean is to eliminate the absolute sign. However, the variance has a disadvantage of having a higher dimension. To reduce it to the same dimension as the data, it is quite natural to use the |

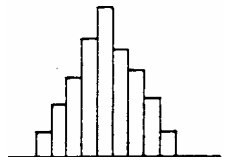
standard deviation(s) $= \sqrt{\frac{1}{n}\sum f\,(x-\bar{x})^2}$

The relation between the graph of the distribution and its standard deviation should be shown.

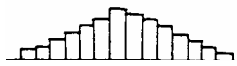
*For example:*



Small value of *s*



Large value of *s*



At this stage, we shall only consider the “spread” of the graphs with reference to the standard deviation.

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 8 | | | | Students should acquire "feeling" for the meaning of mean and standard deviation. This is much more important than expertise in their calculation. Each time they consider a distribution, they may be asked "What percentage of the readings lies within one standard deviation of the mean, within two standard deviations, ...?". We may mention that for many of the distributions we meet in daily life, business and industry, especially those with a bell shape, about $\frac{2}{3}$ of the data lies within one standard deviation from the mean, and almost all within three standard deviations. However, a treatment on normal distribution should not be included. |
| | | 8.4 Method of computing standard deviation (grouped and ungrouped data). | 9 | With certain calculators, we can obtain the value of *s* by simply pressing the *s*-key (or δ-key). However, students should also know how the value of *s* is computed. Obviously, *s* may be computed directly from its definition. An alternative method is to use the following formula: $s=\sqrt{\frac{1}{n}\sum fx^2-\left(\frac{1}{n}\sum fx\right)^2}$ Students are not expected to know how to derive the second formula from the first, but a discussion of the derivation led by teacher will give them faith that the formulae are equivalent. (Teachers should note that the formula used in many calculators for *s* is $s=\sqrt{\frac{1}{n-1}\sum f(x-\bar{x})^2}$ , since it is a better estimate of the standard deviation of the population from which a sample has been taken. Some calculators provide separate keys for calculating *s* using these two different formulae.) |





| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 8 | | 8.5 Application of standard deviation. | 9 | In the teaching of the standard deviation of a distribution, greater emphasis should be placed on the understanding of standard deviation as a measure of dispersion (variability) rather than on the numerical calculation from a complex set of data. Extensive quantitative applications of the standard deviation are not expected but the following examples may be taken as illustrations: *Example* 1 (Standard scores) The standard score $z=\frac{x-\bar{x}}{s}$ is a conversion of raw scores for comparison purposes. Teachers should explain the difference $x-\bar{x}$ and the ratio $\frac{x-\bar{x}}{s}$. The standard score is commonly used in examinations for comparison of students' abilities in different tests. Let us consider the marks in History and Geography of a class of ten students. If a certain student D scores 82 in History and 69 in Geography, in which subject does he do better? |

| | The scores | |
|---|---|---|
| Student | History | Geography |
| A | 95 | 60 |
| B | 90 | 50 |
| C | 80 | 55 |
| D | 82 | 69 |
| E | 79 | 61 |
| F | 60 | 68 |
| G | 70 | 70 |
| H | 85 | 59 |
| J | 68 | 72 |

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 8 | | | | Apparently the student has done relatively better in History. But, if we investigate the scores of the class carefully, we may have a completely different picture. $\bar{x}_1 = 78.4$ $\bar{x}_2 = 63.5$ $s_1 = 9.99$ $s_2 = 7.17$ $z_1 = \frac{82-78.4}{9.99} = 0.36$ $z_2 = \frac{69-63.5}{7.17} = 0.77$ It is natural to assume that the performance of the class is consistent in the two tests. We can then quite reasonably say, from another point of view, that the student does better in Geography than in History. |



*Example* 2 (Life time of electric bulbs)

As a result of tests on electric light bulbs, it was found that the lifetime of a particular make was distributed symmetrically about the mean. The mean lifetime was 2000 hours and the standard deviation was 80 hours. What proportion of bulbs can be expected to have a lifetime

(a) of more than 1 920 hours, and

(b) of more than 2080 hours?

*Example* 3 (Standard deviation as an indication of precision)

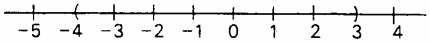
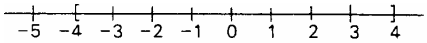
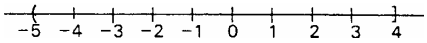
Two instruments, A and B, are used to measure a quantity for the same number of times (20 times with each instrument, say). A gives a standard deviation of 2.6 units while B gives a standard deviation of 1.6 units. Which instrument is more precise?



| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 8 | | | | *Example* 4 (Use of standard deviation to measure non-uniformity) Each of two factories employs the same number of workers. When the monthly salaries of the workers are processed, it is found that the standard deviation of the salaries for the workers of Factory A is \$200 while that of Factory B is \$50. Which factory offers more uniform salaries to its workers? *Example* 5 (Use of standard deviation for setting up acceptable limits) Bags of sugar are filled to the nominal weight of $\mu$ kg by a machine. The actual weights of the sugar in the bags are thus not necessarily equal to $\mu$ kg, but can be somewhat higher or somewhat lower. If a bag weighs much below its nominal value, the customer may claim refund. Usually, the limit for underweight is expressed as the nominal weight minus a certain multiple of the standard deviation (e.g. $\mu - 3\sigma$). Thus, if the nominal weight is 1 kg and the standard deviation is 20 g, the customer may claim refund for a bag of sugar less than 0.94 kg. |

## FORMS IV and V

**Unit 9**

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 9 | **Inequalities**<br><br>*Objectives:*<br>(1) To learn the graphical representation of algebraic inequalities on number lines and on a plane.<br>(2) To learn the solution of algebraic inequalities and to apply it to linear programming problems.<br>(3) To learn the tabulation method to solve quadratic inequalities. | 9.1 The number line and the solution of linear inequalities in one variable. | 4 | This is a revision of the material taught in F. I to F. III. Students should begin with examples like $x > a$ or $x < b$ and then proceed to consider the general inequality $ax + b > c$. Experience shows that it is effective to mark the intervals on the number line by coloured chalk. Inequalities involving "≥" should also be discussed in a similar way. The terms closed and open intervals should be introduced and discussed. There are many ways to mark the closed and open intervals on a number line. The example below indicates one way.<br><br>$-4 < x < 3$  open interval<br>$-4 \le x \le 4$  closed interval<br>$-5 < x \le 4$  half open (or half closed) interval |
| | | 9.2 Quadratic inequalities in one variable. | 4 | A table such as the one below leads easily to the solution of $(x+5)(x-7) < 0$ or $(x+5)(x-7) \ge 0$.<br><br>$x$ … -5 … 7 …<br>$(x + 5)$ – 0 + + +<br>$(x – 7)$ – – – 0 +<br>$(x + 5)(x – 7)$ + 0 – 0 +<br><br>Graphical representation may help students to understand the solutions. |



## FORMS IV and V

**Unit 9**

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 9 | | | | For able students, they can also consider examples like:<br>$(x-1)(x+2)(x+4)(x-7) < 0$ or $\dfrac{(x-1)(x+2)}{(x+4)(x-7)} \ge 0$<br>If they do, the expression on the left should be given in factorized form. |
| | | 9.3 Solution of linear inequalities in two variables. | 7 | It is recommended to introduce the sub-unit from easy in equalities of one variable to more complicated ones in two variables.<br>The following is a possible sequence of examples to be considered:<br><br>(a) $y > c$<br>(b) $y > d$<br>(c) $x > c$ and $y > d$<br>(d ) $x > c$ or $y > d$<br>(e ) $x + y > a$<br>(f) $ax + by > c$<br>Inequalities involving "≥" should also be discussed in a similar way. Graphical representation of solution in each case should be presented neatly on the board and marked with coloured chalk if available. The use of overhead projector, graph board, magnetic graph board or pin board are desirable. In cases where two or three linear inequalities (normally not more than three) are considered, teachers may find it convenient to use the terms "union" and "intersection" informally. |
| | | 9.4 Application to linear programming. | 4 | This demonstrates the use of linear inequalities. School texts are full of graphical examples in linear programming involving two variables. Teachers should try to introduce problems that are more practical and meaningful. A theoretical approach to linear programming is not necessary and discussion should be restricted to graphical treatment. |

## FORMS IV and V

**Unit 10**

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 10 | **Application of trigonometry**<br><br>*Objective:*<br>To apply trigonometric knowledge in solving two and three dimensional problems. | 10.1 True bearings. | 2 | In Form III, students were introduced to the two principal methods of indicating the direction: Compass bearings and True bearings. For calculation at this level, students are expected to use true bearings. Simple problems involving bearings of one point from another or vice versa should be discussed. |
| | | 10.2 Easy problems in two and three dimensions. | 9 | There are many practical problems which involve sine and cosine formulae, both in two and in three dimensions. In particular, problems involving the line of greatest slope would be of interest to students. For three dimensional problems, students should investigate how to solve simple problems involving<br>(a) the angle between two intersecting lines,<br>(b) the angle between a line and a plane, and<br>(c) the angle between two intersecting planes.<br>Only those problems reducible to right-angled triangles are to be considered.<br>Teachers may also find wire-models or 3-D teaching aids useful for explanation and illustration. |
| | | | 11 | |



## FORMS IV and V

**Unit 1**

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 11 | **Coordinate treatment of straight lines and circles**<br><br>*Objectives:*<br>(1) To learn the idea of loci as a basis for further work on simple conics.<br>(2) To discover the relationship of the slopes of parallel lines and that of perpendicular lines.<br>(3) To look at the circle from the coordinate point of view, and to study the equation of circle.<br>(4) To understand the possible intersections between a straight line and a circle. | 11.1 Establishing the concept of locus. | 2 | Approach this idea in as many practical ways as possible e.g. paths of a moving point, a moving line, a moving area and moving objects. |
| | | 11.2 Construction of loci within a plane. | 5 | Construction of the locus of a point moving equidistantly from (a) a fixed point, (b) two fixed points, (c) a fixed line, and (d) two fixed lines.<br>Using simple apparatus such as string, spirograph and mecano-graph, students may construct parabola, ellipse, cycloid and a variety of other loci.<br>The important thing is to select apparatus where the scribe moves according to the given conditions. |
| | | 11.3 Straight line, gradient, parallel and perpendicular lines. | 8 | Revision of $y = mx + c$ emphasizing that the gradient (slope) is also the tangent of the angle θ made with the *x*-axis. Now that tan θ has been defined for the general angle, it is easily demonstrated that $m = \tan\theta$ for θ obtuse as well as acute. Hence, lines are parallel when $m_1 = m_2$ for θ obtuse as well as acute.<br>To demonstrate that for perpendicular lines $m_1 m_2 = -1$ use the theorem about exterior angle of a triangle and $\tan(90° + \theta) = -\frac{1}{\tan\theta}$.<br>Multiple angles should not be used at this stage. The case θ = 90° may be discussed separately.<br>This work used with the mid-point of a line segment opens up further links with other geometry units. Exercises relating to properties of plane figures, such as the diagonals of a parallelogram bisect each other, should give students an awareness of the usefulness of the coordinate system. |

## FORMS IV and V

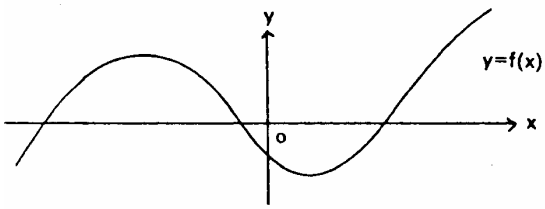
**Unit 11**

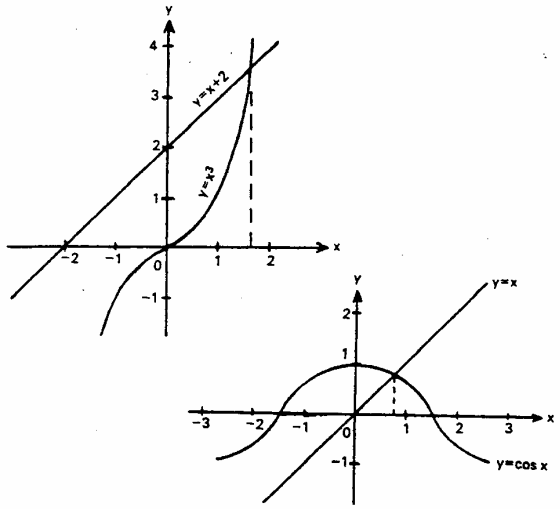
| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 11 | | 11.4 Equation of a circle with centre at the origin. | | 以上的知識，若連綫段中點的性質一併使用，就可與其他幾何單元進一步聯繫起來。有關平面圓形性質的習題，例如平行四邊形的對角綫互相平分，會使學生知道坐標系的用處。 |
| | | 11.5 Equation of a circle in general position. | 3 | 引導學生利用距離公式找出圓心在原點而半徑爲 $r$ 的圓的方程。透過一些例子，學生會發現圓的方程可寫成 $x^2+y^2=r^2$ 。<br>學生也應知道，如果已知方程可寫成上式，那麼它代表一個圓，其圓心在原點而半徑等於 $r$。 |
| | | 11.6 在任意位置的圓的方程。 | 6 | 可與學生討論一般的情況：圓心不在原點而它的坐標是 $(h, k)$。透過一些練習，學生會發現圓的方程可寫成：<br>$x^2+y^2+Dx+Ey+F=0$<br>or $(x-h)^2+(y-k)^2=r^2$ .<br>如果一已知方程可寫成上列形式，學生應知道它代表一個圓，同時知道圓心的位置及半徑的長度。<br>經過不在一直綫上三點的圓的方程，亦應加以討論。 |
| | | 直綫與圓的相交。 | 6 | 直綫與圍是否通常相交於兩點呢？教師應與學生討論所有可能情況，同時應連同二次方程的根一起討論，特別是當二次方程有二重根。幾何中切綫的概念可用代數條件 $b^2-4ac=0$ 來表示。能力較高的學生， |
| | | | 30 | |



## FORMS IV and V

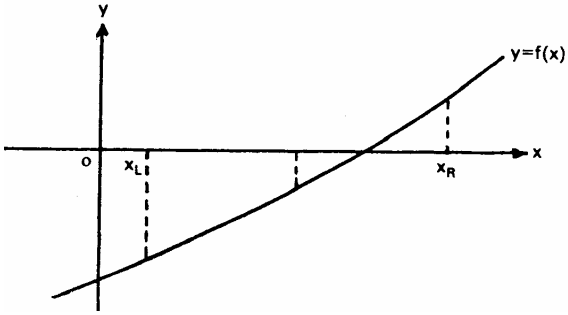
**Unit 12**

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 12 | **Approximate solution of simple equations**<br><br>*Objectives:*<br>(1) To revise and extend the idea of representing equations by graphs.<br>(2) To learn how to solve simple equations by graphical methods.<br>(3) To learn the method of bisection for solving simple equations to a prescribed degree of accuracy. | 12.1 Graphical solution of equations. | | By now, students should be able to solve quadratic equations (with real roots). Taking them a step further, teachers may lead students to consider the solution of other simple equations such as $x=\cos x$ , $x^3-x-2=0$ . Many of these equations cannot be solved algebraically to give exact solutions, but most of them can be solved graphically to give approximate solutions. Teachers should review the graphical representation and solution of quadratic equations studied in Form III (Sub-unit 8.3).<br>Several graphical methods are available for solving simple equations. One method is to arrange the equation in the form $f(x)=0$ . With the help of calculators, it is relatively easy to make a table of values of $f(x)$ for suitable values of $x$ and plot the graph $y=f(x)$ . At a real root of the equation $f(x)=0$ , $y = 0$ and hence the root is the value of $x$ where the graph crosses the $x$-axis and this can be read from the graph.<br>  |

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 12 | | | 5 | Another commonly used method is to arrange the equation in the form $x = g(x)$ and plot the graphs $y = x$ and $y = g(x)$. The points of intersection of these graphs then give the roots of the equation. More generally, we may arrange the equation $f(x) = 0$ in the form $g(x) = h(x)$ and the roots are given by the points of intersection of the curves $y = g(x)$ and $y = h(x)$. Teachers should give an comparison of the methods. Note that answers can be read more easily and more accurately if the curves intersect almost at right angles, and this may serve as one criterion for choosing the graphical method used.   |





| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 12 | | 12.2 Method of bisection. | 10 | While graphical methods work well for many simple equations, they have the disadvantage that the accuracy of the answers cannot be controlled easily. A simple method which can be used to improve the accuracy of the graphical solution and to give the solution to a prescribed degree of accuracy is the method of bisection. In this method, we first find an interval which "brackets" the root and then reduce the "bracketing" interval successively by half until finally the root is "trapped" within an arbitrarily small interval.   |

**Unit 12**

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 12 | | | | For a simple root, a bracketing interval $x_L < x < x_R$ has the property that $f(x_L)$ and $f(x_R)$ have opposite signs, i.e. $f(x_L)f(x_R) < 0$.<br>The teacher may introduce the method by the following example:<br>Find the real root of<br>$x\log x - 1.2 = 0$<br>correct to two decimal places. (The log is to base 10.)<br>A graphical method may be employed to find the first approximation of the root. Alternatively, the following table may serve to find the first bracketing interval:<br><br>$\begin{array}{c\|c} x & f(x) = x\log x - 1.2 \\ \hline 1 & -12 \\ 2 & -0.598 \\ 3 & 0.231 \end{array}$<br><br>$\therefore$ the true root $x_0$ must lie between 2 and 3 and hence we calculate $f(2.5)$ next. |





**Unit 1**

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| 12 | | | | The table below shows the working and should be easy to follow: |

| $x$ | $f(x) = x\log x - 1.2$ | Observation and further step |
|---|---|---|
| 2.5<br>3.0 | –0.205 1<br>0.231 4 | $\therefore$ $2.5 < x_0 < 3.0$<br>Next find $f(2.75)$;<br>$f(2.75) = 0.008\ 2$ |
| 2.5<br>2.75 | –0.205 1<br>0.008 2 | $\therefore$ $2.5 < x_0 < 2.75$<br>$f(2.625) = -0.099\ 8$ |
| 2.625<br>2.750 | –0.099 8<br>0.008 2 | $\therefore$ $2625 < x_0 < 2.750$<br>$f(2.688) = -0.045\ 7$ |
| 2.688<br>2.750 | –0.045 7<br>0.008 2 | $\therefore$ $2.688 < x_0 < 2.750$<br>$f(2.719) = -0.018\ 8$ |
| 2.719<br>2.750 | –0.018 8<br>0.008 2 | $\therefore$ $2.719 < x_0 < 2.750$<br>$f(2.735) = -0.004\ 9$ |
| 2.735<br>2.750 | –0.004 9<br>0.008 2 | $\therefore$ $2.735 < x_0 < 2.750$<br>$f(2.742) = 0.001\ 2$ |
| 2.735<br>2.742 | –0.0049<br>0.001 2 | |

Since $2.735 < x_0 < 2.742$, $x_0 = 2.74$ correct to 2 decimal places.

A sequence of sketches accompanying the steps will illustrate the process still better.

**Unit 1**

| Unit No. | Basic Content/Objectives | Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|---|---|
| | | | | *Notes*<br>(a) At this level, only equations with simple roots will be considered. Equations with equal roots, and more generally, cases where the bisection method does not work, may be discussed qualitatively. Students are not expected to handle such cases themselves.<br>(b) A detailed discussion of the advantages and disadvantages of the bisection method may not be fully appreciated by students at this level. as no other numerical methods have been introduced for comparison. However, after working through several examples, students may realize (i) that the method should work for most of the simple equations, and (ii) a considerable number of iterations may be required to achieve a specified degree of accuracy.<br>(c) The number of iterations required may be calculated as follows: if the initial bracketing interval has width w, then after n iterations, the width will be reduced to ( $\frac{1}{2}$ )$w$. For an accuracy of 2 decimal places, say, this width < 0.01 and so $n > \frac{\log(100w)}{\log 2}$ . |
| | | | 15 | |
| | | Total : | 273 | |